

# Universality of $q_T$ resummation for electroweak boson production<sup>1</sup>

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**Abstract.** We perform a global analysis of transverse momentum distributions in Drell-Yan pair and Z boson production in order to investigate universality of nonperturbative contributions to the Collins-Soper-Sterman resummed form factor. Our fit made in an improved nonperturbative model suggests that the nonperturbative contributions follow universal nearly-linear dependence on the logarithm of the heavy boson invariant mass  $Q$ , which closely agrees with an estimate from the infrared renormalon analysis.

Transverse momentum distributions of heavy Drell-Yan lepton pairs,  $W$ , or  $Z$  bosons produced in hadron-hadron collisions present an interesting example of factorization for multi-scale observables. If the transverse momentum  $q_T$  of the electroweak boson is much smaller than its invariant mass  $Q$ ,  $d\sigma/dq_T$  at an  $n$ -th order of perturbation theory includes large contributions of the type  $\alpha_s^n \ln^m(q_T^2/Q^2)/q_T^2$  ( $m = 0, 1 \dots 2n - 1$ ), which must be summed through all orders of  $\alpha_s$  to reliably predict the cross section [1]. Such resummation is realized in the Collins-Soper-Sterman (CSS) formalism [2], which describes soft and collinear QCD radiation in a wide range of energies by introducing a resummed form factor  $\tilde{W}(b)$  in impact parameter ( $b$ ) space.

While the short-distance contributions ( $b \lesssim 1 \text{ GeV}^{-1}$ ) to the CSS form factor  $\tilde{W}(b)$  can be calculated in perturbative QCD, long-distance nonperturbative contributions from  $b > 1 \text{ GeV}^{-1}$  are not yet fully computable, even though their basic form can be deduced from the infrared renormalon analysis [3]. The factorization theorem behind the CSS formalism predicts that the nonperturbative contributions are universal in unpolarized Drell-Yan-like and semi-inclusive DIS processes. Consequently the function  $\mathcal{F}_{NP}(b, Q)$  that describes the nonperturbative terms can be constrained in a global fit to the hadronic  $q_T$  data, just as the  $k_T$ -integrated parton densities are constrained with the help of inclusive scattering data.  $\mathcal{F}_{NP}(b, Q)$  must be known precisely in order to successfully measure the  $W$  boson mass, because uncertainties in  $\mathcal{F}_{NP}(b, Q)$  may affect the measured value of  $M_W$  at the level comparable to the targeted accuracy of the measurement,  $\delta M_W \approx 30 \text{ MeV}$  at the Tevatron and  $15 \text{ MeV}$  at the LHC. It is therefore interesting to investigate if  $\mathcal{F}_{NP}(b, Q)$  found in the  $q_T$  fit is consistent with the universality hypothesis,

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and whether its preferred form is compatible with the renormalon analysis.

These issues were explored recently in Ref. [4], where a global analysis of  $q_T$  data from fixed-target Drell-Yan pair production and Tevatron Z boson production was performed in the context of an improved model for the nonperturbative contributions. Although  $\mathcal{F}_{NP}(b, Q)$  primarily parametrizes the “power-suppressed” terms, *i.e.*, terms proportional to positive powers of  $b$ , its form found in the fit is correlated with the assumed behavior of the leading-power terms (logarithmic in  $b$  terms) at  $b < 2 \text{ GeV}^{-1}$ . The exact behavior of  $\tilde{W}(b)$  at  $b > 2 \text{ GeV}^{-1}$  is of reduced importance, as  $\tilde{W}(b)$  is strongly suppressed at such  $b$ . For these reasons, we closely followed the procedure of the previous global  $q_T$  analysis [5], while paying close attention to the model of the leading-power terms at perturbative and moderately nonperturbative transverse distances,  $b < 2 \text{ GeV}^{-1}$ .

The large- $b$  contributions were introduced by using the  $b_*$  model [2], as

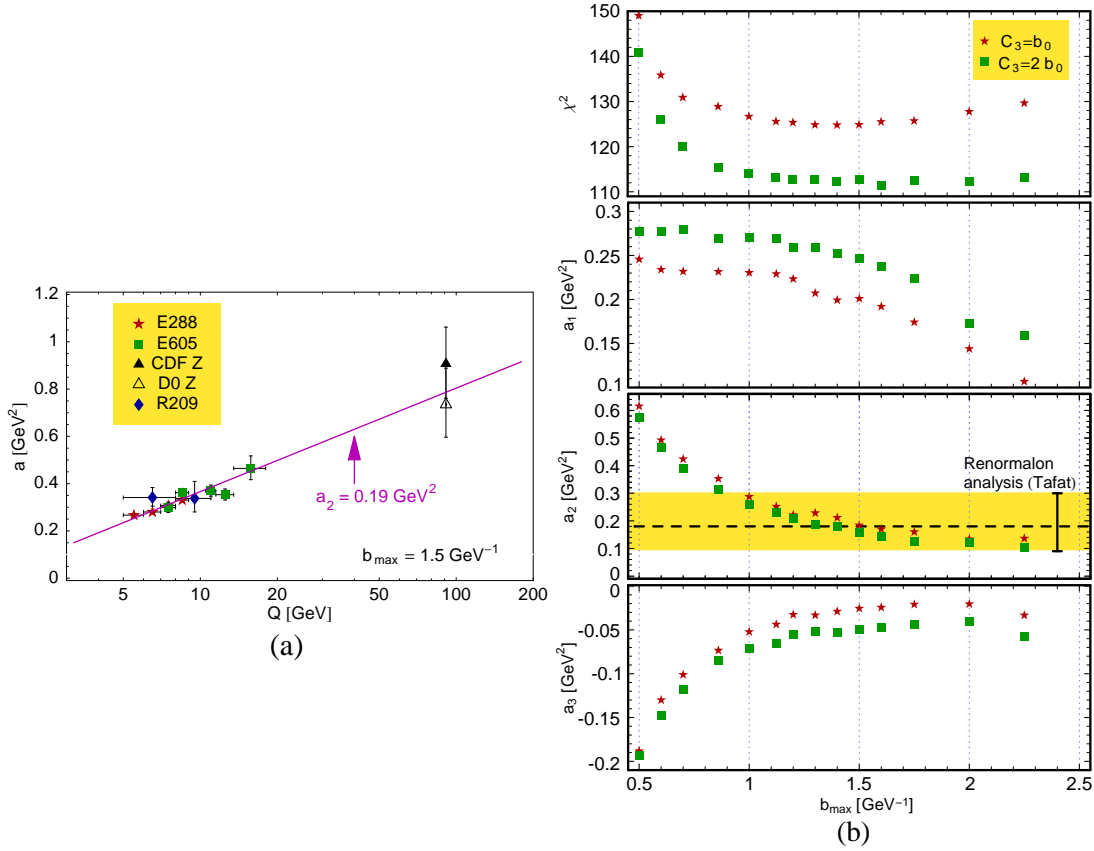
$$\tilde{W}(b) = \tilde{W}_{pert}(b_*) e^{-\mathcal{F}_{NP}(b, Q)}. \quad (1)$$

Here  $\tilde{W}_{pert}(b_*)$  is the perturbative part of  $\tilde{W}(b)$ , *i.e.*, its leading-power part evaluated at a finite order of  $\alpha_s$ .  $\tilde{W}_{pert}(b_*)$  depends on the variable  $b_* \equiv b/(1 + b^2/b_{max}^2)^{1/2}$  and serves as an approximation for all leading-power terms. Its shape is varied at all  $b$  by adjusting a single parameter  $b_{max}$ . The  $b_*$  model with a relatively low  $b_{max} = 0.5 \text{ GeV}^{-1}$  was a choice of the previous  $q_T$  fits [5, 6]. However, it is natural to consider  $b_{max}$  above  $1 \text{ GeV}^{-1}$  in order to avoid *ad hoc* modifications of  $\tilde{W}_{pert}(b)$  in the  $b$  region where perturbation theory is still applicable. In Ref. [4], we proposed a modification in the  $b_*$  model that allowed us to increase  $b_{max}$  at least up to  $\approx 3 \text{ GeV}^{-1}$ , while preserving correct resummation of the large logarithms at small  $b$  and numerical stability of the Fourier-Bessel transform. If a very large  $b_{max}$  comparable to  $1/\Lambda_{QCD}$  is taken,  $\tilde{W}_{LP}(b)$  essentially coincides with  $\tilde{W}_{pert}(b)$ , extrapolated to large  $b$  by using the known, although not always reliable, dependence of  $\tilde{W}_{pert}(b)$  on  $\ln b$ . Hence, the new prescription can be also used to test viability of extrapolation of  $\tilde{W}_{pert}(b)$  to large  $b$ , reminiscent of similar extrapolations introduced in the alternative models [7, 8].

Following the renormalon analysis and Ref. [5], we assumed a Gaussian form of the nonperturbative function,  $\mathcal{F}_{NP}(b, Q) \equiv a(Q)b^2$ , with

$$a(Q) \equiv a_1 + a_2 \ln[Q/(3.2 \text{ GeV})] + a_3 \ln[100x_1x_2]. \quad (2)$$

The dependence of  $\mathcal{F}_{NP}$  on  $\ln Q$  is a consequence of renormalization-group invariance of the soft-gluon radiation. The coefficient  $a_2$  of the  $\ln Q$  term has been related to the vacuum average of the Wilson loop operator and evaluated within lattice QCD as  $0.19^{+0.12}_{-0.09} \text{ GeV}^2$  [9]. To see if the universal Gaussian behavior is consistent with the data, we first examined the values of  $a(Q)$  that are independently preferred by each bin of  $Q$  in 5 examined experimental data sets. Fig. 1(a) shows the best-fit values of  $a(Q)$  obtained in independent fits to the data in each bin of  $Q$  for  $b_{max} = 1.5 \text{ GeV}^{-1}$ . The best-fit  $a(Q)$  follow a nearly linear dependence on  $\ln Q$ , and the slope  $a_2 \equiv da(Q)/d(\ln Q)$  is close to the renormalon analysis expectation of  $0.19 \text{ GeV}^2$  [9]. Such nearly linear behavior of  $a(Q)$  is observed in the entire range  $b_{max} = 1 - 2 \text{ GeV}^{-1}$ , and it less pronounced at  $b_{max}$  outside of the interval  $1-2 \text{ GeV}^{-1}$ . Since the best-fit  $a(Q)$  in each  $Q$  bin are essentially



**Figure 1.** (a) The best-fit values of  $a(Q)$  obtained in independent scans of  $\chi^2$  for the contributing experiments. The vertical error bars correspond to the increase of  $\chi^2$  by unity above its minimum in each  $Q$  bin. The slope of the line is equal to the central-value prediction from the renormalon analysis [9]. (b) The best-fit  $\chi^2$  and coefficients  $a_1$ ,  $a_2$ , and  $a_3$  in  $\mathcal{F}_{NP}(b, Q)$  for different values of  $b_{max}$ . The size of the symbols approximately corresponds to  $1\sigma$  errors for the shown parameters.

independent, we conclude that the data support the universality of  $\mathcal{F}_{NP}$ , when  $b_{max}$  lies in the range  $1 - 2 \text{ GeV}^{-1}$ . In addition, each experimental data set individually prefers a nearly quadratic dependence on  $b$ ,  $\mathcal{F}_{NP} = a(Q)b^{2-\beta}$ , with  $|\beta| < 0.5$  in all experiments.

Next, we performed a simultaneous fit of our model to all the data. Fig. 1(b) shows the dependence of the best-fit  $\chi^2$ ,  $a_1$ ,  $a_2$ , and  $a_3$  on  $b_{max}$ . As  $b_{max}$  is increased above  $0.5 \text{ GeV}^{-1}$  assumed in the studies [5, 6],  $\chi^2$  rapidly decreases, becomes relatively flat at  $b_{max} = 1 - 2 \text{ GeV}^{-1}$ , and grows again at  $b_{max} > 2 \text{ GeV}^{-1}$ . The global minimum of  $\chi^2$  is reached at  $b_{max} \approx 1.5 \text{ GeV}^{-1}$ , where all data sets are described equally well, without major tensions among the five experiments. The magnitudes of  $a_1$ ,  $a_2$ , and  $a_3$  are reduced when  $b_{max}$  increases from  $0.5$  to  $1.5 \text{ GeV}^{-1}$ . In the whole range  $1 \leq b_{max} \leq 2 \text{ GeV}^{-1}$ ,  $a_2$  agrees with the renormalon analysis estimate. The coefficient  $a_3$ , which parametrizes deviations from the linear  $\ln Q$  dependence, is considerably smaller ( $< 0.05$ ) than both  $a_1$  and  $a_2$  ( $\sim 0.2$ ). This behavior supports the conjecture in [7] that  $a_3$  is small if the exact form of  $\tilde{W}_{pert}(b)$  is maximally preserved.

The preference for the values of  $b_{max}$  between  $1$  and  $2 \text{ GeV}^{-1}$  indicates, first, that the data do favor the extension of the  $b$  range where all leading-power terms are ap-

proximated by their finite-order expression  $\tilde{W}_{pert}(b)$ . In  $Z$  boson production, this region extends up to  $3 - 4 \text{ GeV}^{-1}$  as a consequence of the strong suppression of the large- $b$  tail by the Sudakov exponent. The fit to the  $Z$  data is actually independent of  $b_{max}$  within the experimental uncertainties for  $b_{max} > 1 \text{ GeV}^{-1}$ . In the low- $Q$  Drell-Yan process, the continuation of  $b\tilde{W}_{pert}(b)$  far beyond  $b \approx 1 \text{ GeV}^{-1}$  is disfavored because of large higher-order corrections to  $b\tilde{W}_{pert}(b)$  at  $b$  around  $1.5 \text{ GeV}^{-1}$ . To summarize, the extrapolation of  $\tilde{W}_{pert}(b)$  to  $b > 1.5 \text{ GeV}^{-1}$  is disfavored by the low- $Q$  data sets, if a purely Gaussian form of  $\mathcal{F}_{NP}$  is assumed. The Gaussian approximation is adequate, on the other hand, in the  $b_*$  model with  $b_{max}$  in the range  $1 - 2 \text{ GeV}^{-1}$ .

In  $Z$  boson production, our best-fit  $a(M_Z) = 0.85 \pm 0.10 \text{ GeV}^2$  agrees with  $0.8 \text{ GeV}^2$  found in the extrapolation-based models [7, 8], and it is about a third of  $2.7 \text{ GeV}^2$  predicted by the BLNY parametrization. In the low- $Q$  Drell-Yan case, our  $a(Q) = 0.2 - 0.4 \text{ GeV}^2$  is close to the average  $\langle a \rangle = 0.19 - 0.28 \text{ GeV}^2$  in four  $Q$  bins of the E288 and E605 data found in the model [7]. To describe the low- $Q$  data, Ref. [7] allowed a large discontinuity in the first derivative of  $\tilde{W}(b)$  at  $b$  equal to the separation parameter  $b_{max}^{QZ} = 0.3 - 0.5 \text{ GeV}^{-1}$ , where switching from the exact  $\tilde{W}_{pert}(b)$  to its extrapolated form occurs. In the revised  $b_*$  model, such discontinuity does not happen, and  $\tilde{W}_{LP}(b)$  is closer to the exact  $\tilde{W}_{pert}(b)$  in a wider  $b$  range than in Ref. [7].

The best-fit parameters in  $\mathcal{F}_{NP}$  found in the new model are quoted in Ref. [4]. The global fit places stricter constraints on  $\mathcal{F}_{NP}$  at  $Q = M_Z$  than the Tevatron Run-1  $Z$  data alone. Theoretical uncertainties from a variety of sources may be substantial in the low- $Q$  Drell-Yan process, which is indicated, in particular, by the dependence of the agreement with the low- $Q$  data on an arbitrary factorization scale  $C_3$  in  $\tilde{W}_{pert}(b)$ . The low- $Q$  uncertainties do not substantially affect predictions at the electroweak scale. The  $\mathcal{O}(\alpha_s^2)$  corrections and scale dependence are smaller in  $W$  and  $Z$  production, and, in addition, the term  $a_2 \ln Q$ , which arises from the soft factor  $\mathcal{S}(b, Q)$  and dominates  $\mathcal{F}_{NP}$  at  $Q = M_Z$ , shows little variation with  $C_3$ . Consequently, the revised  $b_*$  model with  $b_{max} \approx 1.5 \text{ GeV}^{-1}$  increases our confidence in the transverse momentum resummation at electroweak scales by exposing the soft-gluon origin and universality of the dominant nonperturbative contributions at collider energies.

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